wistful wisdom - Principles of Construction

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Abstract

The three basic principles of this piece are the following:

1. Self similar melody
2. Number system with base 5
3. Mapping of numbers onto musical parameters.

1 Self Similar Melody

A self similar melody is a melody which contains itself in augmentation. This concept has been introduced by Tom Johnson. He gives a nice introduction with an example in the following video: https://www.youtube.com/watch?v=Bqn7cLC6bak. More about the mathematics of calculating such melodies can be found in the following document: https://www.researchgate.net/publication/243044150_Autosimilar_melodies

For "wistful wisdom" the following self similar melody has been used:

$$0, 1, 1, 0, 2, 0, 1, 1, 0, 3, 0, 1, 1, 0, 4, 0, 1, 1, 0, 3, 0, 1, 1, 0, 2$$

This abstract melody has 25 'notes' and has the following property when repeated: If augmented by factor 4 and started 8 notes later there will be always the same notes played together:

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01102011030110401103011020110201103011040110301102011020110301104011030110201
        0   1   1   0   2   0   1   1   0   3   0   1   1   0   4   0   1   1
```

Here the first line will be called the *fast melody* and the second the *slow melody*.

The two melodies are not played synchronously in "wistful wisdom". The fast melody is played in the first quarter of a bar and the slow melody is played in the second quarter. The second quarter is always a pause if the bar number is not from the set $\{9 + 4 \cdot i | i \in \{0, 1, 2, 3, ...\}\}$ because, the slow melody appears only every forth note and it is shifted by 8 notes.

2 Number System with Base 5

In the self similar melody of "wistful wisdom" the number 5 plays an important role:

- It is the number of different 'notes'.
- The length of the self similar melody is a power of 5: $25 = 5^2$.

For this reason the representation of a number in the number system with base 5 has been chosen to be an important source for the mapping. The representation of a number n as a sum of integer multiples of the power of 5 reads:

$$n = \sum_{i=0}^{M} d_i \cdot 5^i = d_M d_{M-1} \cdots d_1 d_0$$

Examples (subscripts indicate the base of the representation): $4_{10} = 4_5$, $7_{10} = 12_5$, $23_{10} = 43_5$, $79_{10} = 304_5$

3 Mapping

The varies parameters of the piece as

- rhythm
- pitches
- instruments
- ways to play

are determined by mapping of numbers onto these parameters.

3.1 Rhythm

Each quarter in the piece is a phrase of up to three micro motives of length 1/16. There are five such micro motives for percussion and tuba:

Percussion:	$0 \rightarrow$, $1 \rightarrow$, $2 \rightarrow$, $3 \rightarrow$, $4 \rightarrow$ tremolo
Tuba:	$0 \rightarrow$, $1 \rightarrow$, $2 \rightarrow$, $3 \rightarrow$, $4 \rightarrow$ 

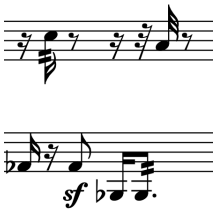
The *bar number* is used to determine which micro motives are used to build a phrase. First a *phrase index* is calculated from the *bar number*:

fast melody: $phrase\,index = bar\,number - 1$
slow melody: $phrase\,index = (bar\,number - 9)/4$

No phrase is defined for the slow melody (second quarter of bar) if the corresponding *phrase index* is not an integer.

The *phrase index* represented in base 5 determines which micro motive is used. The first micro motive is given by a 'note' of the self similar melody by calculating from the two lowest digits an index into this melody. For example, the 'note' at index $14_5 = 9_{10}$ is 3. The second and third micro motives are given by the digits d_2 and d_3, respectively. If the representation of the *phrase index* does not have that many digits the micro motive will be a 16th rest.

Example Bar 233:



Fast melody (first quarter) $phrase index = 233-1 = 232 = 1412_5$. The melody 'note' is 1, which is the eighth note of the self similar melody because $12_5 = 7_{10}$. Thus, the first micro motive is a 16th pause for percussion and a 16th note for tuba. The 2. motive is a 16th note with tremolo for percussion and a 16th pause for tuba because $d_2 = 4$. The 3. motive is again a 16th pause for percussion and a 16th note for tuba because $d_3 = 1$. See the tuba section below in order to understand why it is actually an eighth note.

Slow melody (second quarter) $phrase\,index = (233-9)/4 = 56 = 211_5$. The melody 'note' is 1, which is the seventh note of the self similar melody because $11_5 = 6_{10}$. Thus, the first micro motive is again a 16th pause for percussion and a 16th note for tuba. The 2. motive is a 32nd pause and a 32nd note for percussion and a 16th note for tuba because $d_2 = 2$. Again, see the tuba section below in order to understand why it is actually a 3/8 note. There is no 3. motive because the base-5 representation of the *phrase index* has only three digits.

3.2 Percussion

There are five different types of instruments associated with 'notes' of the self-similar melody:

0 = snare drum
1 = temple blocks
2 = tenor drum

3 = cow bells
4 = wood blocks

The cymbal is handled differently (see below).

Which instrument is played in which bar is given by the *bar number* from which the

$$\textit{instrument index} = \text{floor}((\textit{bar number} - 1)/20) \bmod 25$$

is calculated. The function floor is the largest integer $\leq$ the function argument. This index into the self similar melody gives the corresponding 'note' which is mapped to the instrument by the list above.

Example:

$\textit{bar number} = 190 \Rightarrow \textit{instrument index} = \text{floor}(189/20) \bmod 25 = \text{floor}(9.45) \bmod 25 = 9$

The 'note' at index 9 of the self similar melody is 3. Thus, the percussion instruments in bar 190 are cow bells.

In addition the tenor drum appears after bar 10 in cases in section A, B and G where are longer pauses appear in snare drum or temple blocks. Theses pauses are filled also in accordance with the self-similar melody, using the following mapping:

0 = pause
1 = rub ↕
2 = rub ↕ fast
3 = rub ↻
4 = rub ↕ at rim

The cymbal follows a different schema than the other percussion instruments. For the fast melody the following mapping is applied:

$0 \rightarrow$, $1 \rightarrow$, $2 \rightarrow$, $3 \rightarrow$, $4 \rightarrow$

But instead of the first quarter in a bar it appears in the second one. Also note, that it is not enabled from the beginning but it is gradually enabled towards the end of the piece.

3.3 Tuba

The length of the last note in a phrase is extended such that the phrase is one quarter long.

The last note of every fifth phrase has to be played with tremolo. Every 25. note is played *sf* if *bar number* > 100 (counted separately for slow and fast melody). The frequency is increased to every 5. note for *bar number* > 200.

The pitch for the tuba is calculated as follows:

$$pitch = 28 + 3 \cdot note + \text{floor}(1.5 \cdot \text{floor}(phrase\ index/25))$$

on a chromatic scale where 28 corresponds to E1 and where *note* is the 'note' of the self similar melody at index *phrase index* mod 25.

Example: $bar\ number = 457$

Fast melody $phrase\ index = 457 - 1 = 456 \Rightarrow phrase\ index \bmod 25 = 6 \Rightarrow note = 1 \Rightarrow pitch = 28 + 3 \cdot 1 + \text{floor}(1.5 \cdot \text{floor}(456/25)) = 58 \equiv$ Bb3

Slow melody $phrase index = (457-9)/4 = 112 \Rightarrow phrase index \bmod 25 = 12 \Rightarrow note = 1 \Rightarrow pitch = 28 + 3 \cdot 1 + \text{floor}(1.5 \cdot \text{floor}(112/25)) = 37 \equiv$ C#2